and, for brevity put, $n+n'=t'$, $n+n'+n''=t''$, $n+n'+n''+n'''=t'''$, &c. We shall then have for the value of the annuity multiplied by il_x, putting for I_n, $I_{n'}$, $I_{n''}$ . . . ., in the first terms their values, and multiplying out,

$$\begin{aligned}
&l_x - l_x v^n - cI_n(1+i) + ncv^n \\
&+ l_{x+n}v^n - l_{x+n}v^{t'} - c'I_{n'}v^{n-1} + n'c'v^{t'} \\
&+ l_{x+t'}v^{t'} - l_{x+t'}v^{t''} - c''I_{n''}v^{t'-1} + n''c''v^{t''} \\
&+ l_{x+t''}v^{t''} - l_{x+t''}v^{t'''} - c'''I_{n'''}v^{t'''-1} + n'''c'''v^{t'''} \\
&+ l_{x+t'''}v^{t'''} - l_{x+t'''}v^{t''''} - c''''I_{n''''}v^{t''''-1} + n''''c''''v^{t''''}
\end{aligned}$$

But, since $l_x - l_{x+n} = nc$, $l_{x+n} - l_{x+t'} = n'c'$, &c., and lastly, $l_{x+t'''} = n''''c''''$, we see that the terms $ncv^n - l_x v^n + l_{x+n}v^n$ in the above expression cancel one another. In the same way

$$n'c'v^{t'} - l_{x+n}v^{t'} + l_{x+t'}v^{t'} = 0, \text{ &c.}$$

The value of the annuity will thus be reduced to

$$\frac{1}{il_x}\{l_x - (1+i)(cI_n + c'I_{n'}v^n + c''I_{n''}v^{t'} + c'''I_{n'''}v^{t''} + c''''I_{n''''}v^{t'''})\}.$$

If we replace I_n, I'_n, &c., in this formula, by their equivalents, we shall find for the value of the annuity

$$\frac{1}{i} - \frac{1+i}{i^2 l_x}\{c - (c-c')v^n - (c'-c'')v^{t'} - (c''-c''')v^{t''} - (c'''-c'''')v^{t'''} - c''''v^{t''''}\}.$$

The law of the terms here is evident, and we can at once write down the value of the annuity whatever the number of hypotheses that we choose to make.

If we suppose that there is only one difference, and that it is equal to 1, all the terms $c-c'$, $c'-c''$, &c., vanish; l_x becomes equal to n, and the formula reduces to

$$\frac{1}{i} - \frac{1+i}{i^2 n}(1-v^n)$$

or, replacing $1-v^n$ by its equivalent iI_n,

$$\frac{1}{i} - \frac{1+i}{ni} I_n$$

which is just De Moivre's formula in the simplest form.

On Assurances against the Risk of "Invalidity" or Permanent Inability to Work.

LIFE Assurance provides for the family of the deceased in case of premature death; deferred Annuities provide for old age; but both institutions leave uncovered the risk of premature inability to work.

Invalidity Assurance, including the benefits of a deferred Annuity, would be the real complement to Life Assurance. This truth is so deeply felt in Germany, that a good many institutions, employing a large number of officers, workmen, and labourers; many mills, and particularly the Railway Companies, long since directed their attention to the providing for their officers in case of their being invalided. How were they to calculate the annual contribution, how to make the valuation of their liabilities?

There are no data, or at least very insufficient ones, upon which to calculate for different ages the probability of "becoming an invalid" during the next year; and this want induced Dr. Heym in Leipzic in 1851, when he had to make the computation of Invalidity Annuities for the Leipzic and Dresden Railway Society, to establish the hypothesis that the probability of becoming an invalid during the next year is ·0001 at the age of 20, and 1 at the age of 79, and that it forms a geometrical progression in the interval. The results which he derived from this hypothesis agreed with the observations which Professor Hülsse had made on the Invalidity of miners in Saxony.

Dr. Wiegand accepted this hypothesis as a basis for his calculation of Invalidity Annuities for the Thuringian Railway Societies, published 1859, as "Mathematical foundation of Railway Officers' Invalidity Annuities" (Mathematische Grundlagen für Eisenbahn Pensionscassen), with the single modification that he made the geometrical progression cease at the age of 68, supposing the probability of becoming an invalid equal to 1 at the age of 69. The same hypothesis served as a basis for the Invalidity Annuities of Physicians and for the calculations in Wiegand's work, "Invalidity Assurance" (Versicherung gegen Erwerbsunfähigkeit. Halle, 1865).

But Dr. Wiegand did not stop here. Invalidity Assurance should not remain based upon a mere hypothesis. He made it his object to open the source of experience, which is to furnish sufficiently trustworthy data for the computation of the probability of becoming an invalid. Such data are in possession of the Railway Companies; and at last Dr. Wiegand sees himself, after numerous exertions, in sight of his aim. He avails himself of a moment of repose in his labours to report on what has been achieved and on what is to follow. His survey is contained in a very valuable and interesting work "On Mortality and Invalidity Statistics for Railway Officials" (Die Mortalitäts- und Invaliditäts-Statistik bei Eisenbahn-Beamten Actenmässige Darstellung der darauf bezüglichen Operationen).

After a short historical introduction, relating the facts just mentioned, which induced him to take up the question of the probability of becoming an Invalid, Dr. Wiegand places before the reader all the measures he had to take before he at last succeeded in obtaining the full support of the different Railway Companies which are now ready to furnish the data of their experience. It will be impossible here to enumerate all his steps. I must refer the readers of this Magazine to the just-named work; and I am sure they will not withhold their admiration from the man who by his never flagging zeal and energy, by enormous labour, and by untiring perseverance, at last persuaded the leading members of the Railway Companies of the expediency of his requests, so that they not only furnished some data just at hand, but elected a Committee, of which Dr. Wiegand is a Member, for the full examination of the Invalidity of Railway Officials.

The object of Dr. Wiegand is by no means confined to the law of Invalidity amongst Railway Officials. He is well aware of the importance of Invalidity Assurance in general, and he has no doubt that Life Assurance Offices will willingly cultivate this branch, as soon as they possess the necessary data for its computation. An Invalidity Table of Railway Officials will, in Dr. Wiegand's opinion, enable them to grant Invalidity Assurance to persons in any occupation, as the Invalidity to be expected in other pursuits is less than with Railway Officials, the Railway Service requiring so vigorous a constitution, that it must pension off its Officials, as invalids, in a state of health which would not entitle them to be considered as invalid in another occupation.

It is scarcely necessary to add that Dr. Wiegand, while he puts at the head of his researches the Invalidity, has not neglected the Mortality of Railway Officials. He says (page 4), "It is nearly an axiom with Life "Assurance Societies that railway Officials are subject to a very high rate of "mortality. Constant reports of railway accidents, where so and so many "officials have perished, must create the persuasion that this class has a very "low expectation of life. That is the reason why some Offices entirely "refuse to assure Railway Officials, and others only accept them with a very "considerable extra premium. But there exists no real measure for the "extra risk founded upon experience, and each Office is guided only by the "vague feelings of its manager. This state of things is unreasonable. If "there exists an extra risk, its true value must be found out by observation."

With reference to Mortality, Dr. Wiegand has examined the materials hitherto furnished to him by the Railway Societies; and although the data are by no means numerous enough to establish a general truth, the results of his examination are important and interesting.

Among 10952 Railway Officials under observation during a year, of whom 2193 belonged to the train officials, the mortality has been 124 and 25 respectively. According to the Experience table it ought to have been 134·2179 and 24·528.

Among 73379 Railway Officials under observation 415 cases of Invalidity have occurred, while, according to Dr. Wiegand's Hypothesis, there would have been 401·789 invalids.

The coincidence is surprising, but only an examination based on more numerous facts will decide about the real law.

Hamburg, May, 1869. WILHELM LAZARUS.

Notes on Newton's Formulæ for Interpolation.

I.

NEWTON, in a celebrated lemma (*Princ. Phil. Nat. Math.*, Lib. iii., *Lemma* v.), proposed and solved the fundamental problem of interpolation by finite differences; and thereby, to say the least, gave us the foundation of the theory of differences.

In the *Methodus Differentialis*, printed in 1715, he treats the same subject more at length; but after a careful examination, I cannot but think that this little treatise was written many years before the said lemma, which bears all appearance of being the ripe fruit of his researches on this matter.